

A pupil really understands a mathematical concept, idea or technique if he or she can:

- describe it in his or her own words;
- represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)⁸
- explain it to someone else;
- make up his or her own examples (and non-examples) of it;
- see connections between it and other facts or ideas;
- recognise it in new situations and contexts;

- make use of it in various ways, including in new situations.⁹

Developing mastery with greater depth is characterised by pupils' ability to:

- solve problems of greater complexity (i.e. where the approach is not immediately obvious), demonstrating creativity and imagination;
- independently explore and investigate mathematical contexts and structures, communicate results clearly and systematically explain and generalise the mathematics.

Number and place value	<ul style="list-style-type: none"> • For whole numbers, the more digits a number has, the larger it must be: any 4-digit whole number is larger than any 3-digit whole number. But this is not true of decimal numbers: having more digits does not make a decimal number necessarily bigger. For example, 0.5 is larger than 0.35. Ordering decimal numbers uses the same process as for whole numbers ie we look at the digits in matching places in the numbers, starting from the place with the highest value ie from the left. The number with the higher different digit is the higher number. For example, 256 is greater than 247 because 256 has 5 tens but 247 has only 4 tens. Similarly 1.0843 is smaller than 1.524 because 1.0843 has 0 tenths but 1.524 has 5 tenths.
Addition and subtraction	<ul style="list-style-type: none"> • Deciding which calculation method to use is supported by being able to take apart and combine numbers in many ways. For example, calculating $8.78 + 5.26$ might involve calculating $8.75 + 5.25$ and then adjusting the answer. The associative rule helps when adding three or more numbers: $367 + 275 + 525$ is probably best thought of as $367 + (275 + 525)$ rather than $(367 + 275) + 525$.
Multiplication and division	<ul style="list-style-type: none"> • Standard written algorithms use the conceptual structures of the mathematics to produce efficient methods of calculation. • Standard written multiplication method involves a number of partial products. For example, 36×24 is made up of four partial products 30×20, 30×4, 6×20, 6×4. • There are connections between factors, multiples and prime numbers and between fractions, division and ratios
Fractions	<ul style="list-style-type: none"> • Fractions express a relationship between a whole and equal parts of a whole. Pupils should recognise this and speak in full sentences when answering a question involving fractions. For example, in response to the question 'What fraction of the journey has Tom travelled?' the pupil might respond, 'Tom has travelled two thirds of the whole journey.' • Equivalent fractions are connected to the idea of ratio: keeping the numerator and denominator of a fraction in the same proportion creates an equivalent fraction. • Putting fractions in place on the number lines helps understand fractions as numbers in their own right.
Measurement	<ul style="list-style-type: none"> • To read a scale, first work out how much each mark or division on the scale represents. • The unit of measure must be identified before measuring. Selecting a unit will depend on the size and nature of the item to be measured and the degree of accuracy required.
Geometry	<ul style="list-style-type: none"> • Variance and invariance are important ideas in mathematics, particularly in geometry. A set of quadrilaterals for example may vary in many ways in terms of area, length of sides and the size of individual angles. However there are a set of invariant properties which remain common to all quadrilaterals, namely they have four sides and their internal angles sum to 360°. Some of these properties emerge from naturally occurring constraints, for example the sum of the internal angles will always sum to 360°, they can do nothing else! The questions 'What's the same?' and 'What's different?' can draw pupils' attention to variance and invariance. • Shapes can be alike in essentially two different ways: congruent and similar. Congruent shapes are alike in all ways: they could occupy exactly the same space. Similar shapes share identical geometrical properties but can differ in size. All equilateral triangles are similar, but only identically sized ones are congruent. Not all isosceles triangles are similar. • Angle properties are a mix of necessary conditions and conventions. It is a necessary condition that angles on a straight line combine to a complete half turn. That we measure the half turn as 180° is conventional.
Statistics	<p>Pie charts visually display relative proportions, for example, that the proportion of pupils at School A liking reading is greater than the proportion at School B.</p>