

A pupil really understands a mathematical concept, idea or technique if he or she can:

- describe it in his or her own words;
- represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)<sup>8</sup>
- explain it to someone else;
- make up his or her own examples (and non-examples) of it;
- see connections between it and other facts or ideas;
- recognise it in new situations and contexts;

- make use of it in various ways, including in new situations.<sup>9</sup>

Developing mastery with greater depth is characterised by pupils' ability to:

- solve problems of greater complexity (i.e. where the approach is not immediately obvious), demonstrating creativity and imagination;
- independently explore and investigate mathematical contexts and structures, communicate results clearly and systematically explain and generalise the mathematics.

Number and place value	<ul style="list-style-type: none"> <li>• Imagining the position of numbers on a horizontal number line helps us to order them: the number to the right on a number line is the larger number. So 5 is greater than 4, as 5 is to the right of 4. But <math>-4</math> is greater than <math>-5</math> as <math>-4</math> is to the right of <math>-5</math>.</li> <li>• Rounding numbers in context may mean rounding up or down. Buying packets of ten cakes, we might round up to the nearest ten to make sure everyone gets a cake.</li> <li>• Estimating the number of chairs in a room for a large number of people we might round down to estimate the number of chairs to make sure there are enough.</li> <li>• We can think of place value in additive terms: 456 is <math>400 + 50 + 6</math>, or in multiplicative terms: one hundred is ten times as large as ten.</li> </ul>
Addition and subtraction	<ul style="list-style-type: none"> <li>• It helps to round numbers before carrying out a calculation to get a sense of the size of the answer. For example, <math>4786 - 2135</math> is close to <math>5000 - 2000</math>, so the answer will be around 3000. Looking at the numbers in a calculation and their relationship to each other can help make calculating easier. For example, <math>3012 - 2996</math>. Noticing that the numbers are close to each other might mean this is more easily calculated by thinking about subtraction as difference.</li> </ul>
Multiplication and division	<ul style="list-style-type: none"> <li>• It is important for children not just to be able to chant their multiplication tables but to understand what the facts in them mean, to be able to use these facts to figure out others and to use them in problems.</li> <li>• It is also important for children to be able to link facts within the tables (e.g. <math>5 \times</math> is half of <math>10 \times</math>).</li> <li>• They understand what multiplication means and see division as both grouping and sharing, and to see division as the inverse of multiplication.</li> <li>• The distributive law can be used to partition numbers in different ways to create equivalent calculations. For example, <math>4 \times 27 = 4 \times (25 + 2) = (4 \times 25) + (4 \times 2) = 108</math>.</li> <li>• Looking for equivalent calculations can make calculating easier. For example, <math>98 \times 5</math> is equivalent to <math>98 \times 10 \div 2</math> or to <math>(100 \times 5) - (2 \times 5)</math>. The array model can help show equivalences.</li> </ul>
Fractions	<ul style="list-style-type: none"> <li>• Fractions arise from solving problems, where the answer lies between two whole numbers.</li> <li>• Fractions express a relationship between a whole and equal parts of a whole. Children should recognise this and speak in full sentences when answering a question involving fractions. For example, in response to the question <i>What fraction of the chocolate bar is shaded?</i> the pupil might say <i>Two sevenths of the whole chocolate bar is shaded.</i></li> <li>• Equivalency in relation to fractions is important. Fractions that look very different in their symbolic notation can mean the same thing.</li> </ul>
Measurement	<ul style="list-style-type: none"> <li>• The smaller the unit, the greater the number of units needed to measure (that is, there is an inverse relationship between size of unit and measure).</li> </ul>
Geometry	<ul style="list-style-type: none"> <li>• During this year, pupils increase the range of 2-D and 3-D shapes that they are familiar with. They know the correct names for these shapes, but, more importantly, they are able to say why certain shapes are what they are by referring to their properties, including lengths of sides, size of angles and number of lines of symmetry.</li> <li>• The naming of shapes sometimes focuses on angle properties (e.g. a rectangle is right-angled), and sometimes on properties of sides (e.g. an equilateral triangle is an equal sided triangle).</li> <li>• Shapes can belong to more than one classification. For example, a square is a rectangle, a parallelogram, a rhombus and a quadrilateral.</li> </ul>
Statistics	<p>In mathematics the focus is on numerical data. These can be discrete or continuous. Discrete data are counted and have fixed values, for example the number of children who chose red as their favourite colour (this has to be a whole number and cannot be anything in between). Continuous data are measured, for example at what time did each child finish the race? (Theoretically this could be any time: 67.3 seconds, 67.33 seconds or 67.333 seconds, depending on the degree of accuracy that is applied.) Continuous data are best represented with a line graph where every point on the line has a potential value.</p>